Economics 8103
Microeconomic Theory
Spring 1999

Lecture 10
Implementation and the Revelation Principle

I. Introduction

A. A dominant theme of economics in recent decades has been the role of private information.

B. In general, to the extent that economic outcomes should be responsive to private information, there must be incentives for truthful revelation.

II. Examples of Problems Giving Rise to Incentive Compatibility Constraints

A. If contributions to the cost of a public good are positively related to willingness to pay, there is an incentive to “free ride.”

B. In an exchange economy setting, with agents whose preferences are unknown, there is the possibility of acting as a monopolist or monopsonist.

C. In elections with more than two candidates, voting systems can give rise to incentives to vote “insincerely.”

III. Framework

A. Information

1. $X$ is the set of alternatives.

2. $\{1, \ldots, I\}$ is the set of agents.

3. For each $i = 1, \ldots, I$, $\Theta_i$ is a set of types.

4. $\Theta = \Theta_1 \times \ldots \times \Theta_I$.

5. For each $i = 1, \ldots, I$, $u_i : X \times \Theta \rightarrow \mathbb{R}$ is a utility function.
a. Usually we will consider problems with *private values*, meaning that $u_i$ does not depend $\theta_j$ for $j \neq i$.

6. In the case of Bayesian implementation there is a probability measure on $\Theta$. (In MWG this is represented by a “probability density” $\phi(\cdot)$.)

B. The Game Form

1. A *mechanism of game form* is $\Gamma = (S_1, \ldots, S_I, g(\cdot))$ where

$$g : S_1 \times \ldots \times S_I \rightarrow X.$$ 

IV. Common Informational Assumptions

A. If the agents all know all components of $\theta$, then for each $\theta$ there is an induced game. This leads to the notion of *Nash implementation* of an outcome correspondence $F : \Theta \rightarrow X$.

B. The agents know $\phi(\cdot)$, and each agent knows her own component of $\Theta$, but no one knows more than this. This leads to the notion of *Bayesian Nash implementation* of an outcome function $f : \Theta \rightarrow X$.

C. For a variety of reasons, among them the possibility that the social planner may have no control of how much each agent knows beyond $\phi(\cdot)$ and $\theta_i$, we may be interested in strategies $s_i : \Theta_i \rightarrow S_i$ with the property that $s_i(\theta_i)$ is optimal regardless of the types of the other agents. This leads to the notion of dominant strategy implementation.

V. The Revelation Principle

A. Given pure strategies $s_i : \Theta_i \rightarrow S_i$, there is an induced outcome function

$$\theta \mapsto g(s_1(\theta), \ldots, s_i(\theta)).$$ 

B. For any notion of equilibrium above there are corresponding *incentive compatibility conditions*. 

2
C. Conversely, any outcome function can be interpreted as a revelation mechanism, and in this context equilibrium conditions for truthful revelation amount to incentive comaptability conditions.

VI. A Concrete Example

A. Two agents have to allocate an indivisible object with possible side payments.

B. The spaces of types are $\Theta_1 = \{0, 2\}$ and $\Theta_2 = \{1, 3\}$, where each type represents the agent’s valuation for the object. Assume that all four elements of $\Theta$ have probability one fourth.