Economics 8103
Microeconomic Theory
Spring-1 2005

Problem Set 4
Due: February 17, 2004

Problem 1: (30 pts.) Problem 8.F.2 (p. 265) of MWG.

Problem 2: (30 pts.) Let $N = (S_1, \ldots, S_I, u_1, \ldots, u_I)$ be a normal form game. Prove that the set of perfect equilibria of $N$ is a closed set.

Problem 3: (40 pts.) In an all pay auction the object goes to the high bidder, with a 50-50 coin flip in the event of a tie, and all bidders pay their bids regardless of whether they win. We will study an all pay auction between two bidders who are each known to be willing to pay 1 for the object. Representing the two agents’ strategies by cumulative distribution functions $F_1$ and $F_2$ (or, if you like, by probability measures $\mu_1$ and $\mu_2$ on $[0, 1]$) let $(F_1^*, F_2^*)$ (or $(\mu_1^*, \mu_2^*)$) be a Nash equilibrium.

(a) Prove that in this equilibrium neither agent assigns positive probability to any single positive bid. (Hint: consider the other agent’s best response to a strategy with such a “mass point.”)

(b) Prove that if one agent assigns probability zero to an interval $(a, b)$ where $0 \leq a < b \leq 1$, then the other agent assigns no probability to the interval $(a, b + \varepsilon)$ for some $\varepsilon > 0$.

(c) Prove that both agents assign positive probability to every interval $(a, b)$ where $0 \leq a < b \leq 1$.

(d) Using the fact that each agent is indifferent between all bids between 0 and 1, find the Nash equilibrium.