Problem 1: (33 pts.) Problem 8.B.5 (p. 262) of MWG, modified as follows: (a) argue that there exist numbers \( a', b' > 0 \) such that for any \( q_1, q_2 \), each firm’s profits with the given parameters are what they would be with \( a, b, c \) replaced by \( a', b', 0 \); (b) do the problem as stated with the simplifying assumption that \( c = 0 \).

Problem 1: (33 pts.) Let \( N = (S_1, \ldots, S_I; u_1, \ldots, u_I) \) be a normal form game. Suppose that \( r^1_{j_1}, \ldots, r^\ell_{j_\ell} \) is a sequence of pure strategies that can be iteratively eliminated by strict mixture dominance. For each \( i \) let

\[
\hat{S}_i := S_i \setminus \{ r^h_{j_h} : h = 1, \ldots, \ell, j_h = i \}.
\]

Prove that for each \( h = 1, \ldots, \ell \), there is some \( \sigma_{j_h} \in \Delta(\hat{S}_{j_h}) \) such that \( u_{j_h}(\sigma_{j_h}, t_{-j_h}) > u_{j_h}(r^h_{j_h}, t_{-j_h}) \) for all \( t_{-j_h} \in \hat{S}_{-j_h} \).

Problem 3: (34 pts.) Let \( S_1, \ldots, S_I \) be nonempty finite sets of pure strategies, and let \( S := S_1 \times \cdots \times S_I \). Let

\[
\mathcal{E} : (\mathbb{R}^S)^I \to \Sigma := \Delta(S_1) \times \cdots \times \Delta(S_I)
\]

be the *Nash equilibrium correspondence*: for \( u = (u_1, \ldots, u_I) \in (\mathbb{R}^S)^I \), \( \mathcal{E}(u) \) is the set of Nash equilibria of the game \((S_1, \ldots, S_I, u_1, \ldots, u_I)\). Prove that \( \mathcal{E} \) is upper semicontinuous.