I. Introduction

A. In honor of the 2014 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel, we will study the paper “Using Cost Observation to Regulate Firms” by Jean-Jacques Laffont and Jean Tirole.

1. This illustrates how mechanism design can be applied to somewhat a variety of practical settings.


B. The setting is that a government regulator is tasked with arranging a contract for the provision of a public good.

1. The quality $q$ of the good and the total cost $C$ are directly observable.

2. Quality $q$ produces consumer surplus $S(q)$ where $S$ is a function with $S(0) = 0$, $S' > 0$, and $S'' < 0$.

3. Total accounting cost is $C = (\beta - e)q + \varepsilon$ where $\beta \in [\underline{\beta}, \overline{\beta}]$ is initial marginal cost, $e$ is the level of effort, and $\varepsilon$ is a random variable with mean zero.

   a. The firm observes $\beta$, then chooses $e$, then observes the realization of $\varepsilon$. The regulator does not observe these variables.

   b. The firm’s disutility of effort is $\psi(e)$.
i. This does not enter the accounting measure of cost \( C \), perhaps because it is an intangible such as “morale.”

D. A contract specifies a total payment \( t + C \) to the firm, where \( t = t(q, C) \) is a function of the publicly observed variables.

1. The expected utility of the firm is \( U = E(t) - \psi(e) \).
2. The social cost of this payment is \( (1 + \lambda)(t + C) \) where \( \lambda \geq 0 \) is a parameter reflecting either the excess burden caused by taxation or perhaps a belief that payments to the firm increase inequality.
3. Expected social welfare is

\[
W(q, e, U) = S(q) + U - (1 + \lambda)(t + C) = S(q) - (1 + \lambda)(t + (\beta - e)q - U) - \lambda U
\]

\[
= S(q) - (1 + \lambda)(\psi(e) + (\beta - e)q) - \lambda U.
\]

II. Full Information Solution

A. In the absence of unobservable information, the regulator would choose \( q, e, \) and \( t \) to maximize \( W(q, e, U) \) subject to \( U \geq 0 \). (The last condition is individual rationality for the firm.)

B. The first order necessary conditions for the problem are:

1. \( U = 0 \).
2. \( S'(q) = (1 + \lambda)(\beta - e) \).
3. \( \psi'(e) = q \).

III. The following assumptions insure that a full information solution with positive \( q \) exists for all \( \beta \in [\underline{\beta}, \overline{\beta}] \):

A. \( S'(0) > \overline{\beta} \).

1. It is never optimal to produce nothing.

B. We assume that \( \psi : [0, \hat{e}) \to [0, \infty) \) is \( C^1 \), where \( \hat{e} < \beta \), and that \( \psi(0) = 0, \psi'(e) > 0 \) and \( \psi''(e) > 0 \) for all \( e \geq 0 \), and \( \lim_{e \to \hat{e}} \psi(e) = \infty \).
1. For some results Laffont and Tirole assume that $\psi''/\psi'$ is nonincreasing. This implies that $\ln(\psi'(e)) \leq me + b$ for $m$ and $b$, and is easily seen to be inconsistent with the assumptions above.

C. $\lim_{q \to \infty} S'(q) < \beta - \hat{e}$.

2. Welfare cannot be increased by increasing $q$ without limit.

D. $\psi'(0) < (S')^{-1}(\beta)$.

1. At the smallest $q$ that might be efficient, some effort in reducing cost is desirable.

E. $S''\psi'' + 1 + \lambda < 0$.

1. The determinant of the matrix of second partial derivatives of $W(\cdot, \cdot, U)$ is $-(1 + \lambda)S''\psi'' - (1 + \lambda)^2$, so this inequality (together with $S'' < 0$ and $\psi'' > 0$) implies that this matrix is negative definite, so that $W(\cdot, \cdot, U)$ is strictly differentiably concave.

F. These assumptions differ from Laffont and Tirole’s in several ways. (This aspect of the paper is a bit sloppy.) The overall point is to insure that we only need to consider solutions that are interior, which they do throughout, so their analysis is valid for our assumptions.

IV. The Optimal Incentive Scheme

A. The variables and functions.

1. By the revelation principle, a scheme in which the transfer is a function of realized quality and cost can be replaced by one in which the firm announces an initial cost $\beta$, after which the regulator stipulates a quality $q(\beta)$ and the transfer is a function $t(\beta, C)$ of the announcement and cost.

2. Let $\hat{\beta}$ denote the actual initial cost.

3. Let $e(\beta)$ be the optimal (for the firm’s utility) effort when $\hat{\beta} = \beta$, given $q(\beta)$ and the transfer function $t(\beta, \cdot)$.

4. Let $C(\beta) = (\beta - e(\beta))q(\beta)$ be the expected cost.
5. Let $s(\beta) = E[t(\beta, C(\beta) + \epsilon)]$ be the expected net transfer.

6. We will simply assume that all functions are as differentiable as need be.
   The paper justifies this.

B. The concealment set.

1. Given actual initial cost $\hat{\beta}$, the concealment set is

   $\{ (\beta, \tau(\beta|\hat{\beta})) : \beta \in [\underline{\beta}, \overline{\beta}], 0 \leq \tau(\beta|\hat{\beta}) = e(\beta) + \hat{\beta} - \beta < \hat{\epsilon} \}.$

   a. In words, the concealment set for $\hat{\beta}$ is the set of pairs $(\beta, e)$ that achieve actual marginal cost $\hat{\beta} - e$ equal to the marginal cost $\beta - e(\beta)$ that is implicitly promised when the firm announces $\beta$.

2. If $\epsilon$ is always zero, then the regulator can detect deviations outside the concealment set, because they result in costs that are different from what is expected.

3. We first study the incentive problem when only deviations within the concealment set are allowed.

4. Incentive compatibility requires that for any given $\hat{\beta}$, setting $\beta = \hat{\beta}$ must maximize $U(\beta|\hat{\beta}) = s(\beta) - \psi(\tau(\beta|\hat{\beta}))$.

5. We compute that

   $$\frac{\partial U}{\partial \beta}(\beta|\hat{\beta})|_{\beta=\hat{\beta}} = s'(\hat{\beta}) - \psi'(\tau(\beta|\hat{\beta}))(e'(\beta) - 1)$$

   and

   $$\frac{\partial^2 U}{\partial \beta \partial \beta}(\beta|\hat{\beta}) = \psi''(\tau(\beta|\hat{\beta}))(e'(\beta) - 1).$$

6. The first order condition is

   $$\frac{\partial U}{\partial \beta}(\beta|\hat{\beta})|_{\beta=\hat{\beta}} = s'(\hat{\beta}) - \psi'(\tau(\beta|\hat{\beta}))(e'(\hat{\beta}) - 1) = 0.$$ 

7. Since $\frac{\partial U}{\partial \beta}(\beta|\hat{\beta})|_{\beta=\hat{\beta}} = 0$ for all $\hat{\beta}$, the second order condition is $e'(\hat{\beta}) \leq 1$:

   $$0 \geq \frac{\partial^2 U}{\partial \beta^2}(\beta|\hat{\beta})|_{\beta=\hat{\beta}} = -\frac{\partial^2 U}{\partial \beta \partial \beta}(\beta|\hat{\beta})|_{\beta=\hat{\beta}} = \psi''(e(\hat{\beta}))(e'(\hat{\beta}) - 1).$$
8. As $\beta$ decreases, quality increases, so $e'(\hat{\beta}) \leq 1$ says that average costs decrease as quality increases.

C. The Regulator’s Problem

1. We assume that the regulator’s prior belief is that $\hat{\beta}$ is uniformly distributed in $[\underline{\beta}, \overline{\beta}]$.
2. Replacing $\hat{\beta}$ with $\beta$, and abusing notation by letting $U(\beta) = s(\beta) - \psi(e(\beta))$, the first order condition is

$$U'(\beta) = s'(\beta) - \psi'(e(\beta))e'(\beta) = -\psi'(e(\beta)).$$

3. We study the optimization problem

$$\max_{q(\cdot), e(\cdot), U(\cdot)} E \int_{\underline{\beta}}^{\overline{\beta}} \left( S(q) - (1 + \lambda)[\psi(e) + (\beta - e)q + \varepsilon] - \lambda U \right) d\beta$$

subject to $U' = -\psi'(e)$ and $U(\beta) \geq 0$ for all $\beta$.

   a. This problem does not include the second order necessary conditions for incentive compatibility. We will need to study whether it is satisfied at a potential solution.
   b. Since $\psi' > 0$, if $U(\overline{\beta}) \geq 0$, then $U(\beta) \geq 0$ for all $\beta$.
   c. Note that since $\varepsilon$ has mean zero, the problem reduces to

$$\max_{q(\cdot), e(\cdot), U(\cdot)} \int_{\underline{\beta}}^{\overline{\beta}} \left( S(q) - (1 + \lambda)[\psi(e) + (\beta - e)q] - \lambda U \right) d\beta$$

subject to $U' = -\psi'(e)$ and $U(\overline{\beta}) \geq 0$.

4. This is an optimal control problem with state variable $U(\beta)$ and control variables $q(\beta)$ and $e(\beta)$. We will develop the necessary conditions for an interior solution intuitively rather than rigorously.

   a. At each $\beta$, taking $e(\beta)$ and $U(\beta)$ as given, we should set $q(\beta)$ to maximize the integrand, giving the first order condition

$$S'(q) = (1 + \lambda)(\beta - e). \quad (*)$$
b. Consider increasing $e$ by $\Delta e$ in the interval $[\beta, \beta + \Delta \beta]$. The approximate effect of the change of the integrand in this interval is

$$(1 + \lambda)(q - \psi')\Delta e \Delta \beta.$$ 

The state variable $U(\beta)$ at $\beta$ will be larger than it would have been by approximately $\psi'' \Delta e \Delta \beta$, and this will increase the expense of providing the required utility to the firm by $\lambda(\beta - \beta)\psi'' \Delta e \Delta \beta$. Equating benefits and costs gives the first order condition

$$\psi'(e) = q - \frac{\lambda}{1 + \lambda}(\beta - \beta)\psi''(e)$$  (**).  

5. Henceforth we assume that there is a unique interior solution.

V. Implementation

A. Equations (*) and (**) may be regarded as a system of equations that determine, for each $\beta$, the levels of output and effort $q^*(\beta)$ and $e^*(\beta)$.

1. We set $U^*(\beta) = \int_{\beta}^{\beta} \psi'(e^*(\delta)) d\delta$. The expected transfer net of accounting cost is then $s^*(\beta) = U^*(\beta) + \psi(e^*(\beta))$.

2. We should check that this satisfies the second order necessary condition $\frac{d e^*}{d \beta} \leq 1$ for the firm’s choice of effort to be optimal. Unfortunately this depends on techniques from the theory of optimal control (and is also a mess) so we skip it.

B. To implement this solution, we need to find a transfer function $t(\beta, C)$ that induces the desired effort, by which we mean that no deviations (including those going outside the concealment set) are improving for the firm.

1. Let

$$t(\beta, C) = s^*(\beta) + K^*(\beta)[C^*(\beta) - C]$$

where $K^*(\beta) = \frac{\psi'(e^*(\beta))}{q^*(\beta)}$. 

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2. Given this transfer function and actual $\hat{\beta}$, the firms problem is to choose $\beta$ and $e$ to maximize

$$E[U] = E[K^*(\beta)(C^*(\beta) - C) + s^*(\beta) - \psi(e)]$$

$$= E[K^*(\beta)(C^*(\beta) - (\hat{\beta} - e)q^*(\beta) - \varepsilon) + s^*(\beta) - \psi(e)]$$

$$= K^*(\beta)(C^*(\beta) - (\hat{\beta} - e)q^*(\beta)) + s^*(\beta) - \psi(e)$$

$$= s^*(\beta) - \psi'(e^*(\beta))\bar{e}'(\beta|\hat{\beta}) + \psi'(e^*(\beta))e - \psi(e).$$

3. The first order conditions for maximization amount to there being no infinitesimal deviations in the concealment set that are improving, and also no variations $e$ for the given $\beta$ that are improving.

a. The FOC with respect to $e$ gives $\psi'(e) = \psi'(e^*(\beta))$ and thus $e = e^*(\beta)$.

b. Taking account of this, the firms problem is to choose $\beta$ to maximize

$$s^*(\beta) - \psi'(e^*(\beta))\bar{e}'(\beta|\hat{\beta}) + \psi'(e^*(\beta))e^*(\beta) - \psi(e^*(\beta)).$$

c. The first order condition is

$$0 = s^{*\prime}(\beta) - \psi''(e^*(\beta))e^{*\prime}(\beta)\bar{e}'(\beta|\hat{\beta}) - \psi'(e^*(\beta))\frac{\partial \bar{e}'}{\partial \beta}(\beta|\hat{\beta})$$

$$+ \psi''(e^*(\beta))e^{*\prime}(\beta)e^*(\beta) + \psi'(e^*(\beta))e^{*\prime}(\beta) - \psi'(e^*(\beta))e^{*\prime}(\beta)$$

$$= s^{*\prime}(\beta) - \psi''(e^*(\beta))e^{*\prime}(\beta)\bar{e}'(\beta|\hat{\beta}) - \psi'(e^*(\beta))\frac{\partial \bar{e}'}{\partial \beta}(\beta|\hat{\beta})$$

$$+ \psi''(e^*(\beta))e^{*\prime}(\beta)e^*(\beta).$$

d. Taking into account $\frac{\partial \bar{e}'}{\partial \beta}(\beta|\hat{\beta}) = e'(\beta) - 1f$ and the first order condition $(\dagger)$, we see that this is satisfied by setting $\beta = \hat{\beta}$.

4. The second order condition for optimization is a matter of the negative definiteness of the matrix of second partials of the objective function. Laffont and Tirole assert that “straightforward computations” show that this is equivalent to $e^{*\prime}(\beta) \leq 0$. 

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a. Because the firm’s problem is less constrained, it makes sense that this condition should be more demanding than the earlier necessary condition $e'(\beta) \leq 1$.

VI. Unobservable Costs

A. Earlier Baron and Myerson had studied the regulation problem when the cost is unobservable.

B. Now a contract specifies a quality $q$ and a gross transfer to the firm.

1. Suppose a contract $(q(\beta), s(\beta))$ is specified for each $\beta \in [\underline{\beta}, \overline{\beta}]$.
2. Given $\hat{\beta}$, the firm’s problem is to choose $\beta$ and $e$ to maximize

$$U(\beta|\hat{\beta}) = s(\beta) - (\hat{\beta} - e)q(\beta) - \psi(e).$$

3. The first order conditions are

$$s'(\beta) = (\hat{\beta} - e)q'(\beta) \quad \text{and} \quad q(\beta) = \psi'(e).$$

4. Because the firm bears all costs, the second of these is to be expected.

C. Incentive compatibility implies that the first order conditions are satisfied with $\hat{\beta} = \beta$ with $e$ equal to the optimized $e(\beta)$:

$$s'(\beta) = (\beta - e(\beta))q'(\beta) \quad \text{and} \quad q(\beta) = \psi'(e(\beta)).$$

1. Using this, we compute that the derivative of optimized utility is

$$U'(\beta) = s'(\beta) - (1 - e'(\beta))q(\beta) - (\beta - e(\beta))q'(\beta) - \psi'(e(\beta))e'(\beta)$$

$$= s'(\beta) - q(\beta) - (\beta - e(\beta))q'(\beta) = -q(\beta).$$

D. Ignoring the second order conditions, we imagine the planner to be solving the problem

$$\max_{q(\cdot), e(\cdot)} \int_{\underline{\beta}}^{\overline{\beta}} \left[ S(q) - (1 + \lambda)[\psi(e) + (\beta - e)q] - \lambda U \right] d\beta$$

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subject to $U'(\beta) = -q(\beta)$ and $U(\beta) = 0$.

1. This is an optimal control problem with state variable $U(\beta)$ and control variables $q(\beta)$ and $e(\beta)$.
   
a. At each $\beta$, taking $q(\beta)$ and $U(\beta)$ as given, we should set $e(\beta)$ to maximize the integrand, giving the first order condition
   $$\psi'(e) = q.$$

b. Consider increasing $q$ by $\Delta q$ in the interval $[\beta, \beta + \Delta \beta]$. The approximate effect of the change of the integrand in this interval is
   $$[S'(q) - (1 + \lambda)(\beta - e)]\Delta q \Delta \beta.$$

   The state variable $U(\beta)$ at $\beta$ will be larger than it would have been by approximately $\Delta q \Delta \beta$, and this will increase the expense of providing the required utility to the firm by $\lambda(\beta - \beta)\Delta q \Delta \beta$. Equating benefits and costs gives the first order condition
   $$S'(q) = (1 + \lambda)(\beta - e) + \lambda(\beta - \beta).$$

VII. Qualitative Results

A. In comparison with the full information allocation, optimal regulation using cost information leads to lower output and lower effort.

1. Recall that the FOC’s for the full information case are $U = 0$, $S'(q) = (1 + \lambda)(\beta - e)$, and $\psi'(e) = q$.

2. The FOC’s with cost observability are $S'(q) = (1 + \lambda)(\beta - e)$ and
   $$\psi'(e) = q - \frac{\lambda}{1 + \lambda}(\beta - \beta)\psi''(e).$$

3. Comparison of the second pair of conditions shows that $\psi'(e)$ will be smaller using cost information, so that $e$ is smaller, after which the first condition implies that $S'$ will be greater, so that $q$ is smaller.
B. A similar comparison of FOC’s shows that in comparison with the full information case, quality will be lower when costs are unobserved. Since the effort choice is efficient in both cases, effort will be lower.

C. There does not seem to be a clear comparison of the qualities and efforts in the cases of cost observability and unobservability. Of course welfare is higher with observability, because the policies without observability are available to the regulator when costs can be observed.

D. The possibility of implementing the optimal outcome with a linear contract (fixed payment plus partial reimbursement of accounting costs) is an important finding.

1. Such contracts are simple and realistic.

2. Assuming risk neutrality, such contracts are optimal regardless of the distribution of $\varepsilon$. They are the unique contracts with this feature.