I. Introduction

A. In the principal-agent problem there is no asymmetric information at the beginning, but the principal and the agent anticipate that there will be some later.

1. This can arise either because the agent takes some action that the principal cannot observe, or because the agent learns something.

B. Anticipating this, the principal and the agent need to design a contract that does as well as possible, taking the agent’s incentives into account.

II. Moral Hazard

A. In this case the agent will take an action that affects the probabilities of various outcomes.

1. We will follow Jehle and Reny, viewing the action as the degree of care that an insured consumer exerts.

2. This model is more commonly applied to employment.

B. After the insurance contract is in place, the agent will choose an effort level \( e \).

1. For the most part the interesting issues can all be illustrated in a simple setting in which \( e \) is either 0 or 1.

   a. The disutility of effort level \( e \) is \( d(e) \), where \( d(1) > d(0) \).

2. The set of possible losses is \( \{0, 1, \ldots, L\} \), and \( \pi_\ell(e) \) is the probability of loss \( \ell \) given effort \( e \).

   a. We assume that \( \pi_\ell(e) > 0 \) for all \( \ell \) and both \( e \).
b. Of course \( \sum_{\ell=0}^{L} \pi_{\ell}(0) = \sum_{\ell=0}^{L} \pi_{\ell}(1) = 1. \)

3. An insurance contract specifies a premium \( p \) and a benefit \( B_\ell \) for each \( \ell = 0, 1, \ldots, L. \)
   
   a. The benefit \( B_0 \) because can be folded into the premium (adjusting the other \( B_\ell \) appropriately) but this formulation is more symmetric.

4. The consumer’s utility is \( u(v) - d(e) \) where \( u \) is a differentiable increasing strictly concave function of final wealth \( v. \)

5. Let \( w \) be the consumer’s initial wealth.

6. The consumer’s expected utility from this contract is

\[
-d(e) + \sum_{\ell=0}^{L} \pi_{\ell}(e)u(w - p - \ell + B_\ell).
\]

C. Observable Effort

1. For the sake of comparison, and to get a clear initial view, we first study the problem when the insurance company can observe the consumer’s effort.

   a. We interpret this as the insurance company being able to choose \( e. \)

2. The insurer’s problem is

\[
\max_{e,p,B_0,\ldots,B_L} p - \sum_{\ell} \pi_{\ell}(e)B_\ell \quad \text{subject to} \quad -d(e) + \sum_{\ell} \pi_{\ell}(e)u(w - p - \ell + B_\ell) \geq \bar{u}
\]

where \( \bar{u} \) is the consumer’s reservation utility.

3. The Lagrangean is

\[
\mathcal{L} = p - \sum_{\ell} \pi_{\ell}(e)B_\ell - \lambda(\bar{u} + d(e) - \sum_{\ell} \pi_{\ell}(e)u(w - p - \ell + B_\ell)).
\]

4. The first order conditions are \( \lambda \geq 0 \) and

\[
0 = \frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda \sum_{\ell} \pi_{\ell}(e)u'(w - p - \ell + B_\ell),
\]

\[
0 = \frac{\partial \mathcal{L}}{\partial B_\ell} = -\pi_{\ell}(e) + \lambda \pi_{\ell}(e)u'(w - p - \ell + B_\ell),
\]
\[ 0 \geq \frac{\partial L}{\partial \lambda} = \pi + d(e) - \sum_{\ell} \pi_{\ell}(e)u(w - p - \ell + B_\ell). \]

a. The last must hold with equality if \( \lambda > 0 \).

b. The first of these is redundant, because it can be obtained by summing the others.

c. Since \( \pi_{\ell}(e) \neq 0 \), we have \( \lambda > 0 \) and \( u'(w - p - \ell + B_\ell) = 1/\lambda \) for all \( \ell \).

d. Thus \(-\ell + B_\ell\) is the same for all \( \ell \), which means that the contract provides full insurance.

5. Which level of insurance is optimal can be assessed by computing the solution for each \( e \).

D. Unobservable Effort

1. Now we imagine that the insurance company has a target effort level \( e \), and must arrange the customer’s incentives to make this effort choice preferable to the other one, which we denote by \( e' \).

2. Now the insurer’s problem is

\[
\max_{e, p, B_0, \ldots, B_L} p - \sum_{\ell} \pi_{\ell}(e)B_\ell \text{ subject to } -d(e) + \sum_{\ell} \pi_{\ell}(e)u(w - p - \ell + B_\ell) \geq \pi \\
\text{and } -d(e) + \sum_{\ell} \pi_{\ell}(e)u(w - p - \ell + B_\ell) \geq -d(e') + \sum_{\ell} \pi_{\ell}(e')u(w - p - \ell + B_\ell).
\]

3. If the insurer judges that it is optimal to induce \( e = 0 \), this can be accomplished using the policy that is optimal when effort is observable: offer full insurance at the highest premium that the consumer is willing to pay.

   a. Since the consumer is fully insured, there is no reward for exerting effort.

4. When the insurer wishes to induce \( e = 1 \), the Lagrangean is

\[
\mathcal{L} = p - \sum_{\ell} \pi_{\ell}(1)B_\ell - \lambda(\pi + d(1) - \sum_{\ell} \pi_{\ell}(1)u(w - p - \ell + B_\ell)) \\
-\beta(-d(0) + \sum_{\ell} \pi_{\ell}(0)u(w - p - \ell + B_\ell) + d(1) - \sum_{\ell} \pi_{\ell}(0)u(w - p - \ell + B_\ell)).
\]
5. The first order conditions are \( \lambda \geq 0 \), \( \beta \geq 0 \), and

\[
0 = \frac{\partial L}{\partial p} = 1 - \sum_{\ell} (\lambda \pi_{\ell}(1) + \beta (\pi_{\ell}(1) - \pi_{\ell}(0))) u'(w - p - \ell + B_{\ell}),
\]

\[
0 = \frac{\partial L}{\partial B_{\ell}} = -\pi_{\ell}(1) + (\lambda \pi_{\ell}(1) + \beta (\pi_{\ell}(1) - \pi_{\ell}(0))) u'(w - p - \ell + B_{\ell}),
\]

\[
0 \geq \frac{\partial L}{\partial \lambda} = \bar{u} + d(1) - \sum_{\ell} \pi_{\ell}(1) u(w - p - \ell + B_{\ell}),
\]

\[
0 \geq -d(0) + \sum_{\ell} \pi_{\ell}(0) u(w - p - \ell + B_{\ell}) + d(1) - \sum_{\ell} \pi_{\ell}(0) u(w - p - \ell + B_{\ell}).
\]

6. As before, the first is redundant, and the last two hold with equality if \( \lambda \neq 0 \) and \( \beta \neq 0 \) respectively.

7. We rewrite the second as

\[
\frac{1}{u'(w - p - \ell + B_{\ell})} = \lambda + \beta \left[ 1 - \frac{\pi_{\ell}(0)}{\pi_{\ell}(1)} \right].
\]

8. We claim that \( \beta \neq 0 \) and \( \lambda \neq 0 \).

   a. If \( \beta = 0 \), then the equation above implies that \( \ell + B_{\ell} \) does not depend on \( \ell \), but in this case the last of the first order conditions reduces to \( 0 \geq d(1) - d(0) \), which is false.

   b. If \( \pi_{\ell}(0) = \pi_{\ell}(1) \) for all \( \ell \), then \( \lambda > 0 \) because the left hand side of the equation above is positive. If \( \pi_{\ell}(0) \neq \pi_{\ell}(1) \) for some \( \ell \), then (because \( \sum_{\ell} \pi_{\ell}(0) = \sum_{\ell} \pi_{\ell}(1) = 1 \)) there is some \( \ell \) such that \( \pi_{\ell}(0) > \pi_{\ell}(1) \), and again \( \lambda > 0 \).

9. It cannot be the case that \( \pi_{\ell}(0) = \pi_{\ell}(1) \) for all \( \ell \), because then \( \ell - B_{\ell} \) would not depend on \( \ell \) and the last first order condition becomes \( 0 \geq -d(0) + d(1) \), which is false.

10. From the equation above we see that the rate of underinsurance \( \ell - B_{\ell} \) is greater when \( \pi_{\ell}(0)/\pi_{\ell}(1) \) is greater. This is intuitive: the most efficient way to provide incentives is to punish the losses that most strongly suggest lack of effort.
11. We say that the *monotone likelihood ratio property* holds if \( \pi_\ell(0)/\pi_\ell(1) \) is an increasing function of \( \ell \). If this is the case, then the equation above implies that \( \ell - B_\ell \) is an increasing function of \( \ell \).

12. If low effort is optimal when effort is observable, then it is also optimal when effort is unobservable. If high effort is optimal when effort is observable, then it may be optimal when effort is unobservable, or it could be that the cost of providing incentives for high effort is excessive.

III. Hidden Information

A. We now consider a problem in which the agent will learn something after the contract is settled

1. We follow Section 14.C of MWG.

B. The Model

1. The agent’s effort is \( e \in [0, \infty) \), and profits are a deterministic function \( \pi(e) \) of \( e \).
   
   a. We assume that \( \pi \) is increasing, \( C^1 \), and strictly concave.

2. The agent’s utility is

\[
u(w, e, \theta) = v(w - g(e, \theta))
\]

where \( w \) is the wage, \( \theta \) is state of nature, and \( g(e, \theta) \) is the monetary value of the disutility of effort.

a. The two possible values of \( \theta \) are \( \theta_L \) and \( \theta_H \). We assume that \( g(e, \theta_L) > g(e, \theta_H) \) and that \( \frac{\partial g}{\partial e}(e, \theta_L) > \frac{\partial g}{\partial e}(e, \theta_H) \), so that \( g(e, \theta_L) > g(e, \theta_H) \) for all \( e > 0 \).

b. We assume that \( v \) is increasing, \( C^1 \), and strictly concave, and that \( v'(w) \to 0 \) as \( w \to \infty \).

c. We assume that for each \( \theta = \theta_L, \theta_H \), \( g(\cdot, \theta) \) is an increasing, \( C^1 \), strictly convex function.
c. [Give additional conditions on \( v \) and \( g \).]

3. The agent’s reservation utility is \( \bar{u} \). He is allowed to quit at any time, so he must receive at least this utility in both states.

C. The State is Observable

1. Let \((w^*_L, e^*_L)\) and \((w^*_H, e^*_H)\) be the contract that maximizes profits, subject to the reservation utility constraint, when the state can be observed.

2. We assume that each of these is interior, in the sense that \(0 < e^*_L, e^*_H < \infty\).

3. If the reservation utility constraints were not satisfied with equality, then the principal could increase profits by decreasing the wage, so

\[ v(w^*_L - g(e^*_L, \theta_L)) = \bar{u} \quad \text{and} \quad v(w^*_H - g(e^*_H, \theta_H)) = \bar{u}. \]

a. For any \((w, e)\) we have \(v(w - g(e, \theta_H)) > v(w - g(e, \theta_L))\), so

\[ v(w^*_L - g(e^*_L, \theta_H)) > v(w^*_L - g(e^*_L, \theta_L)) \]

and

\[ v(w^*_H - g(e^*_H, \theta_L)) < v(w^*_L - g(e^*_L, \theta_L)). \]

4. In both cases a small increase in \( e \) increases \( \pi \) in proportion to the derivative of \( \pi \), and it increases the wage bill in proportion to \( \frac{\partial g}{\partial e} \), so we have

\[ \pi'(e^*_L) = \frac{\partial g}{\partial e}(e^*_L, \theta_L) \quad \text{and} \quad \pi'(e^*_H) = \frac{\partial g}{\partial e}(e^*_H, \theta_H). \]

5. It must be the case that \( e^*_H > e^*_L \), because if \( e_H \leq e^*_L \), then \( \pi'(e_H) \geq \pi'(e^*_L) \) and \( \frac{\partial g}{\partial e}(e_H, \theta_H) \leq \frac{\partial g}{\partial e}(e^*_L, \theta_H) < \frac{\partial g}{\partial e}(e^*_L, \theta_L) \).

D. The Principal Cannot Observe the State

1. If \( p \) is the probability that \( \theta = \theta_H \), then the principal’s problem is

\[ \max_{w_L, e_L, w_H, e_H \geq 0} (1 - p)[\pi(e_L) - w_L] + p[\pi(e_H) - w_H] \]

subject to
(a) \( v(w_L - g(e_L, \theta_L)) \geq \pi \),
(b) \( v(w_H - g(e_H, \theta_H)) \geq \pi \),
(c) \( v(w_L - g(e_L, \theta_L)) \geq v(w_H - g(e_H, \theta_L)) \),
(d) \( v(w_H - g(e_H, \theta_H)) \geq v(w_L - g(e_L, \theta_H)) \).

2. Note that (b) is a consequence of (a) and (d):

\[ v(w_H - g(e_H, \theta_H)) \geq v(w_L - g(e_L, \theta_H)) \geq v(w_L - g(e_L, \theta_L)) \geq \pi. \]

3. Let \((\hat{w}_L, \hat{e}_L)\) and \((\hat{w}_H, \hat{e}_H)\) be a solution.

4. If \( v(\hat{w}_L - g(\hat{e}_L, \theta_L)) > \pi \), then it would be possible to lower both \( \hat{w}_L \) and \( \hat{w}_H \) be some small \( \varepsilon > 0 \) without violating (a), (c), or (d), so (a) must hold with equality.

5. We must have \( \hat{e}_H \geq \hat{e}_L \), because if \( \hat{e}_H < \hat{e}_L \), then \( v(\hat{w}_H, g(\hat{e}_H, \theta_H)) \geq v(\hat{w}_L, g(\hat{e}_L, \theta_H)) \) would imply that \( v(\hat{w}_H, g(\hat{e}_H, \theta_L)) \geq v(\hat{w}_L, g(\hat{e}_L, \theta_L)) \) (because \( \frac{\partial g}{\partial e}(e, \theta_L) > \frac{\partial g}{\partial e}(e, \theta_H) \)) contrary to (c).

6. In fact \( \hat{e}_H > \hat{e}_L \), because if \( \hat{e}_H = \hat{e}_L \), then \( \hat{w}_H = \hat{w}_L \), and it would be possible to increase profits either by moving \((\hat{w}_L, \hat{e}_L)\) to the left along the indifference curve given by (a), or by moving \((\hat{w}_H, \hat{e}_H)\) to the right along the indifference curve given by (d).

7. We have \( \hat{e}_H = e_H^* \), because for fixed \((\hat{w}_L, \hat{e}_L)\) the principal is adjusting \((w_H, e_H)\) along an indifference curve given by (d), and the first order condition is the same as in the problem with observable state.

8. We have \( \hat{e}_L < e_L^* \). At any point \((w_L, e_L)\) to the right of \( e_L^* \) on the indifference curve given by (a), moving to the left along the indifference curve increases profits when \( \theta_L \) is the state, and diminishes the wage that must be paid to induced \( e_H^* \) when the state is \( \theta_H \). At \( \hat{e}_L = e_L^* \) the first effect vanishes, but the second effect is positive, so the optimum must be further to the left.