Economics 6030
Microeconomics B
Second Semester

Lecture 7
Signalling

I. Introduction

A. So far we have seen how private information may affect market behavior in two contexts.

1. In the case of the lemons market, the decision was binary, namely either sell the car or keep it.

2. In the case of the insurance market, it was possible to achieve a more sophisticated response because both the premium and the benefit could be adjusted.

B. In this lecture we consider two settings in which an agent may try to take actions in response to private information in preparation for the market.

1. In Spence’s model of signalling, educational credentials are accepted as evidence of ability because they are less costly for high ability individuals.

2. In the model of statistical discrimination developed by Arrow and Phelps, individuals’ efforts in preparing for the market are affected by their expectations, and the market forms expectations concerning preparation based on observable characteristics, so if there are multiple equilibria, one group may be stuck in an inefficient equilibrium.
II. Spence’s Model of Credentialing

A. The set of possible worker types is $\Theta = \{\theta_L, \theta_H\}$, where $0 < \theta_L < \theta_H$.

1. The types are identified with the worker’s marginal product.
   a. For simplicity, and to demonstrate that the signalling effect does not depend on the signalling activity being productive or otherwise meritorious, we assume that education has no effect of productivity.

2. The probability that a worker has type $\theta_H$ (or the fraction of the population with type $\theta_H$) is $\lambda$.

B. Each worker chooses an educational level $e \in [0, \infty)$.

1. Let $c_L(e)$ and $c_H(e)$ be the costs of education level $e$ for the two types. We assume that:
   a. $c_L(0) = c_H(0) = 0$.
   b. $c_L'(e) > c_H'(e) > 0$ for all $e$.
   c. MWG assume that $c_L''(e), c_H''(e) > 0$ for all $e$, but in fact the whole model is invariant under monotonic rescalings of the education variable, and (at least as far as I can tell) it is enough to assume that $c_L(e), c_H(e) \to \infty$ as $e \to \infty$.

2. The payoffs to the two types of worker resulting from a level of education $e$ and a wage $w$ are

   $$w - c_L(e) \quad \text{and} \quad w - c_H(e).$$

C. After observing the worker’s choice of $e$, two firms engage in a first price sealed bid auction for the worker.

1. We assume that both firms have the same beliefs after seeing any $e$.
   a. In spirit, if not in technical detail, this is consistency in the sense of sequential equilibrium.

2. For an education level chosen with positive probability, this belief is required to be Bayesian, but otherwise it is unrestricted.
3. Competition between the firms drives the wage to the expected productivity, given the belief.

III. Types of Equilibria.

A. Let \( \tilde{e} \) and \( e_1 \) be the levels of education such that, respectively,

\[
\theta_H - c_L(\tilde{e}) = \theta_L \quad \text{and} \quad \theta_H - c_H(e_1) = \theta_L.
\]

1. For any \( e \in [\tilde{e}, e_1] \) there is a separating equilibrium.
2. When \( e > \tilde{e} \) this equilibrium is constrained inefficient.

B. For \( w \in ((1-\lambda)\theta_L + \lambda \theta_H, \theta_H) \) let \( e_w \) be the education level such that \( w - c_L(e_w) = \theta_L \).

1. For such a \( w \) and associated \( e_w \) there is a semi-separating equilibrium in which the high quality worker always chooses \( e_w \) and the low quality worker mixed between 0 and \( e_w \).

C. Let \( e' \) be such that

\[
\theta_L = (1 - \lambda)\theta_L + \lambda \theta_H - c_L(e').
\]

1. For any \( e \in [0, e'] \) there is a pooling equilibrium in which all workers choose \( e \) and receive \( (1 - \lambda)\theta_L + \lambda \theta_H \).
2. These equilibria can easily dominate the separating equilibria.

D. The only equilibrium that satisfies the intuitive criterion is the separating equilibrium in which the high skilled types choose \( \tilde{e} \) and receive wage \( \theta_H \) while the low skilled types choose 0 and receive \( \theta_L \).

IV. Imperfectly Observed Productive Education and Discrimination

A. In the first period the worker chooses whether to make an investment that cannot be observed directly, and which has cost \( c \).

1. If there is no investment, the probability that the worker becomes “qualified” is 0.01.
2. If there is investment, the probability that the worker becomes qualified is 0.8.

B. In the second period the workers are tested.
   1. Qualified workers “pass” with probability 0.9.
   2. Unqualified workers pass with probability 0.1.

C. After the test is observed, the worker receives her expected wage.

D. For some values of $c$ there are three equilibria, one in which all workers invest, another in which no workers invest, and a third (presumably unstable) mixed equilibrium.

E. If different groups in the population differ according to some observable characteristic, they may experience different equilibria, even if the characteristic has no ex ante significance.