I. Introduction

A. One of the most famous and influential papers in the history of economics is “The Market for Lemons” by George Akerlof.

1. Imagine a world with potential sellers and buyers of used cars.
2. If a car is “good,” then the seller’s reservation price is $4000 while any buyer would be willing to pay $5000.
3. If a car is a “lemon,” then it is worth $2000 to potential buyers and $1000 to its owner.

B. An equilibrium consists of a price $p$ for used cars and decisions on the part of the agents concerning whether to sell or buy such that:

1. No owner of a car sells it if she receives less than it is worth to her.
2. No owner of a car fails to sell it if $p$ is greater than its value to her.
3. The price $p$ does not exceed the expected value of the cars that are sold on the market.
4. No buyer fails to buy a car if the price is less than the expected value of cars that are sold.

C. The possibilities for equilibrium depend on the relative numbers of buyers and sellers, and the ratio of good cars to bad cars.
1. First suppose that there are more potential buyers than sellers.
   a. All cars can be transacted if the expected value of a car to a buyer exceeds $4000, and in this equilibrium the price is the expected value.
   b. Even when there are many good cars, there is also an equilibrium in which \( p = 2000 \) and only lemons are traded.
   c. If the expected value of a car to a buyer is less than $4000, then only lemons are traded.
2. Now suppose there are many more potential sellers than buyers.
   a. If the expected value of a car exceeds $4000, then there is an equilibrium in which \( p = 4000 \), and all potential buyers obtain a car.
   b. There is also an equilibrium in which \( p = 1000 \) and only lemons are traded.
   c. When the expected value of a car is less than $4000, this is the only equilibrium.
3. If the number of potential buyers exceeds the number of owners of low quality cars, but is less than the total number of car owners, then there is an equilibrium with price \( p = 2000 \), and if there are enough good cars, there may also be an equilibrium with \( p = 4000 \).

C. Now suppose that a lemon is worth $2000 to its owner and $1000 to a buyer.
1. If there are enough good cars, there may still be an equilibrium in which both good cars and lemons are traded.
2. There is always an equilibrium in which no cars are traded, and this is the only equilibrium if the fraction of lemons is sufficiently high.

D. We will examine a more elaborate version of this model, but one should remember this particular example, as it is what comes to economists’ minds.
when “lemons” is used as a technical term.

II. The Model

A. Workers’ types are distributed in $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$.

1. The c.d.f. of the distribution is $F(\cdot)$, with associated density $f$.
2. We interpret the worker’s type $\theta$ as the worker’s productivity in employment.
3. Let $r(\theta)$ denote the reservation wage of a worker of type $\theta$. We interpret this as the productivity of the worker in home production.
4. In the simple Akerloff model above, the workers are the potential sellers of used cars, and the distribution consists of a mass point at $\underline{\theta}$ and a mass point at $\overline{\theta}$.
5. Another important application is health insurance, in which $\theta$ is the individual’s health status (roughly, the negation of expected claims) and $r(\theta)$ is the negation of the premium at which an individual with health status $\theta$ is indifferent about whether to purchase insurance.

B. For wage $w$ let $\Theta(w) := \{ \theta | r(\theta) \leq w \}$ be the set of types that would accept employment.

1. In an equilibrium with employment the wage $w$ satisfies the zero profit condition

   $$ w = \mathbb{E}[\theta | \theta \in \Theta(w)]. $$

2. There is an equilibrium in which there is no employment if $\Theta(\theta) < \emptyset$.
3. From a technical point of view the key assumption is that $\mathbb{E}[\theta | \theta \in \Theta(w)]$ is a continuous function of $w$.
   a. By the intermediate value theorem, if there is some $w$ such that $\mathbb{E}[\theta | \theta \in \Theta(w)] > w$, then there is an equilibrium in $(w, \infty)$.
4. Let $w^*$ be the maximum competitive equilibrium wage.
III. Possible Types of Equilibria.

A. It can happen that the only equilibrium has no employment, even though all workers are more productive on the job than at home.

1. For example, suppose that \( r(\theta) = \theta - \varepsilon(\theta - \theta) \). Everyone is more productive working, but it can easily happen that at any wage greater than \( \theta \), the people who accept that wage are not productive enough to justify it.

B. There may be multiple equilibria, as we have already seen, and in fact there can be an arbitrary number of equilibria.

1. These are Pareto ranked according to the wage, since all workers are weakly better off with a higher wage, and in all equilibria firms make zero profits.

2. In MWG it is argued that the low wage equilibria may be unstable, since, by offering a higher wage, a firm may attract an applicant pool whose improved quality is at least sufficient to compensate for the increase in the wage bill.

   a. They model this as a two stage game.

      i. In the first stage two firms simultaneously post wages.

      ii. In the second stage each worker decides whether to work for one of the two firms or stay home.

      iii. If there is some \( \varepsilon > 0 \) such that \( E[\theta|\theta \in \Theta(w)] > w \) for all \( w \in (w^* - \varepsilon, w^*) \), then the unique subgame perfect equilibrium has both firms posting \( w^* \).

3. This is one example of the concept of an *efficiency wage*, which means that by paying a higher wage the employer does better than with a low wage.

4. The model is not rich enough to fully explain brokers who receive fees
in return for facilitating profitable matches, but it does illustrate the potential gains from trade from such activities.

IV. Constrained Efficiency

A. In a model in which agents possess private information, an allocation is incentive compatible if no agent can do better by pretending to have a different type than she actually has.

1. In the model studied here, incentive compatibility requires that all agents who work receive the same wage, and all agents who do not work receive the same unemployment benefit.

B. An important conceptual lesson of the economics of information is that it is at least slightly misleading to describe an outcome as “inefficient” in comparison with what would be possible in a world with complete information, since policy makers typically do not have access to that world.

C. The more relevant notion is constrained efficiency, meaning Pareto optimality within the set of allocations that satisfy the incentive compatibility constraints.

1. It turns out that if \( r(\theta) \) is a strictly increasing function of \( \theta \), then the \( w^* \) equilibrium is constrained efficient in the sense that, even when the policy maker is able to dictate both a wage \( w_e \) and an unemployment benefit \( w_u \), subject to a nonnegative net revenue constraint, there is no possibility of Pareto improvement.

   a. In order to obtain a contradiction, suppose that everyone is at least as well off, and some are strictly better off, and total consumption does not exceed total production, when the government mandates \( w_e \) and \( w_u \).

   b. If everyone works in the \( w^* \) equilibrium, then the outcome
is unconstrained optimal, hence Pareto optimal by the first welfare theorem.

c. If everyone works when $w_e$ and $w_u$ is mandated, then all receive $w_e$, which must be the average product. If no one is worse off than before, then each $\theta$ is receiving at least $r(\theta)$. This amounts to full employment being an equilibrium, which is necessarily the $w^*$ equilibrium.

d. Therefore we may assume that, under $w_e$ and $w_u$, some person is not working. Since that person cannot be worse off than before we must have $w_u \geq 0$.

e. If, under $w_e$ and $w_u$, no one works, then each $\theta$ must consume $r(\theta)$, because that’s all there is. Since this is an option in the $w^*$ equilibrium, and no one is worse off, this outcome must coincide with the $w^*$ equilibrium.

f. Therefore we may assume that, under $w_e$ and $w_u$, some person works. Since that person cannot be worse off we must have $w_e \geq w^*$.

g. If $w_e - w_u \leq w^*$, then fewer people will be working than in the $w^*$ equilibrium. Since the ones that drop out of the labor force are the ones with highest $r(\theta)$, and thus also highest $\theta$, the average output of workers is less than $w^*$, and the average wage bill is $w_e \geq w^*$, which is impossible.

i. Without the assumption that $r$ is strictly increasing, it can happen that providing a positive $w_u$ can allow $w_e > w^*$ because people with low $\theta$ and high $r(\theta)$ are induced to stay home.

h. If $w_e \geq w^* + w_u$, then the total production of those work-
ing must be sufficient to fund their consumption, and also the unemployment benefit. In this case the intermediate value theorem would imply the existence of an equilibrium with a wage higher than $w_e \geq w^*$, contrary to assumption.

V. The Rat Race

A. Up to this point the agent with private information has only two options, which limits the complexity of the analysis.

B. The following model is from Akerlof’s “The Economics of Caste and the Rat Race and Other Woeful Tales” (Quarterly Journal of Economics 1976).

1. There are classes of workers $n = 1, \ldots, N$.
2. There are assembly lines with speeds $S = 0, 1, 2, \ldots$.
3. A worker of class $n$ who works on an assembly line of speed $S$ and consumes $G$ has utility

$$U_n = G - S - \frac{3}{8}(S - n)^2,$$

so up to some point higher class workers prefer working faster, and even after that point they tolerate it more easily.
4. The output per worker of an assembly line of speed $S$ is $Q = \bar{\pi} + S$ where $\bar{\pi}$ is the average class of the workers.
5. There is no shortage of capital, so workers are paid the average product of their assembly line, and equilibrium has a no-profitable-entry condition.

C. In equilibrium workers of class 1 work on an assembly line of speed 1, and workers of class $n = 2, \ldots, N$ work on assembly lines of speed $n + 1$.

1. For a worker of class $n$, where $3 \leq n \leq N - 1$, working on an assembly line of speed $n + 1$ gives consumption of $2n + 1$ and speed disutility of $n + 1 \frac{3}{8}$, so utility is $n - \frac{3}{8}$.
a. Working on an assembly line of speed \( n \) gives consumption of 
\( 2n - 1 \) and speed disutility of \( n \), so utility is \( n - 1 \).

b. Working on an assembly line of speed \( n + 2 \) gives consumption of 
\( 2n + 3 \) and speed disutility of \( n + \frac{7}{2} \), so utility is \( n - \frac{1}{2} \).

2. For a worker of class \( N \) the only difference is that moving up to an 
assembly line of speed \( N + 2 \) does not increase the average quality of 
coworkers, so this cannot be advantageous.

3. For a worker of class 2, it cannot be advantageous to move up to an 
assembly line of speed 4 for the reasons given above. It is also not 
advantageous to move down to an assembly line of speed 1.

   a. Working on an assembly line of speed 3 gives a consumption of 
   5 and a disutility of speed of \( 3\frac{3}{8} \), so utility is \( 1\frac{5}{8} \).

   b. Working on an assembly line of speed 1 gives a consumption of 
   2 and a disutility of speed of \( 1\frac{2}{8} \), so utility is \( \frac{5}{8} \).

4. For a worker of class 1, working on an assembly line of speed 1 gives 
a consumption of 2 and a disutility of speed of 1, so utility is 1.

   a. Working on an assembly line of speed 0 gives a consumption of 
   1 and a disutility of speed of \( \frac{3}{8} \), so utility is \( \frac{5}{8} \).

   b. Working on an assembly line of speed 3 gives a consumption of 
   5 and a disutility of speed of \( \frac{9}{2} \), so utility is \( \frac{1}{2} \).

5. We need to check that assembly lines of speed 2 cannot enter.

   a. Workers of class 1 will be attracted if the wage exceeds \( 3\frac{3}{8} \).

   b. Workers of class 3 will be attracted if the wage exceeds \( 3\frac{5}{8} \).

   c. Provided that there are enough class 1 workers, relative to the 
   number of class 2 workers, entry is not possible.

D. This equilibrium is not optimal in the unconstrained sense, since each class 
of worker would be better off if they could work on an assembly line of speed
equal to their class, with other workers of their class.

1. Suppose that the government imposes a tax on assembly lines of one per worker per unit of speed, then the equilibrium would have all workers of class \( n \) working on an assembly line of speed \( n \), and receiving a wage of \( n \).

   a. Moving up to an assembly line of speed \( n + 1 \) increases consumption by 1 and increases the disutility of speed by \( \frac{3}{8} \).

   b. Moving down to an assembly line of speed \( n - 1 \) decreases consumption by 1 and decreases the disutility of speed by \( \frac{5}{8} \).

   c. A lump sum redistribution of the tax proceeds gives rise to an unconstrained Pareto optimum, and a much more equal distribution of income.

2. This argument does not show that the equilibrium is not constrained optimal, and at this point I do not know whether it is if, as Akerlof assumes, all classes have the same population.

VI. The Rothschild-Stiglitz Insurance Model

A. The issue in this model is screening, which means that the set of contracts that are offered to the consumer are designed to elicit revelation of the underlying type.

   1. Tradition emphasizes a distinction between screening and signalling, in which agents with private information take observable actions prior to the market.

   2. We will see such a model, but it is good to bear in mind that these are two simple examples of how private information can interact with markets, among many others.

B. To begin with we consider a single consumer who faces a probability \( \pi \) of
having an accident that incurs a financial loss of $L$.

1. An insurance policy consists of a premium $p$ that is paid whether or not the accident occurs, and a benefit $B$ that is paid in the event of the accident.

2. The insurance policy is actuarially fair if $\pi B = p$.

3. The consumer has wealth $w$ in the event of no accident, and a $C^2$ increasing strictly differentiably concave utility function $u$, which depends on final wealth.

   a. The expected utility after buying the insurance policy is

   $$\pi \cdot u(w - p - L + B) + (1 - \pi) \cdot u(w - p).$$

   b. If the consumer can buy any quantity of actuarially fair insurance, she will fully insure, so that the final wealth in the two states in the same.

C. The problem arises when the consumer knows which of two possible accident probabilities $\pi_H$ and $\pi_L$ she has.

1. Let $\alpha$ be the fraction of the population with accident probability $\pi_L$.

2. Many of the results below depend on there being only two types. For this reason one should bear in mind that in actual insurance markets these problems can be more complicated.

D. Following Jehle and Reny, we model this as an extensive form game.

1. The stages of play are:

   a. Nature chooses the type of the consumer.

   b. The consumer, who knows her type, proposes an insurance policy to the insurer.

   c. The insurer, who sees the proposed policy, but does not directly observe the consumer’s type, accepts or rejects the policy.
2. We define a *sequential equilibrium* to be a policy proposal for each type of consumer, a function $\beta$ from policy proposals to $\Delta(\{H,L\})$ specifying the beliefs of the insurer after each proposal, and a function from policy proposals to \{Accept, Reject\} for the insurer, such that:
   
   a. For the actual policies chosen by the consumer, the insurer’s beliefs are given by Bayesian updating.
   b. For each policy proposal, the insurer accepts or rejects according to whether the premium exceeds the expected payout, given the beliefs.
   c. The policy proposal of each type of consumer is optimal, given the response function of the insurer.

3. This notion departs from the notion of sequential equilibrium for finite extensive form games in several ways.
   
   a. Consistency is expressed implicitly. Since the consumer has infinitely many actions, there are some technical issues, but not real difficulties, as the right definition will amount to the conditions we are requiring.
   b. We are not allowing the consumer to mix over more than one policy proposal. For the most part this is just a natural simplification.

E. Equilibrium analysis.

1. The equilibrium utility of each type is bounded below by the optimal policy among those that are actuarially fair for $\pi_H$.
   
   a. For type $H$, this means full insurance, which we denote by $\psi^*_H$.
   b. In reality there are many different risk types, so instead of a lower bound there can be a *death spiral* in which premiums increase as only higher and higher risk types purchase insurance,
until the market collapses.

2. In a *separating equilibrium* type $H$ proposes $\pi^*_H$, while type $L$ proposes a policy that is at least as attractive as that to her, but not more attractive than $\psi^*_H$ to $H$, and which is no worse than actuarially fair for the insurer relative to $\pi_L$.

   a. In response to any policy other than these, the insurer believes that the type is $H$ with probability one, and accepts only if the policy is profitable relative to that belief.

3. In a *pooling equilibrium* there is a single policy that is proposed by both types, and which is accepted with sufficient probability to deter deviations to policies that will certainly be accepted.

   a. For each type this must be better than the best policy that is actuarially fair relative to $\pi_H$.

   b. It must be no worse than actuarially fair for the insurer, relative to the accident probability $\alpha \pi_L + (1 - \alpha) \pi_H$.

   c. Again, in response to any policy other than these, the insurer believes that the type is $H$ with probability one.

4. In the original paper, though not in the textbooks we are using, there is the notion of a *semiseparating equilibrium* in which one type makes only one type of proposal while the other type mixes between that proposal and a proposal that only she makes.

F. Cho and Kreps’ Intuitive Criterion

1. As with many games with private information, the set of sequential equilibria is large because it is possible to deter out-of-equilibrium actions by specifying extremely unfavorable beliefs in response.

2. Cho and Kreps argue that a belief that the deviator is type $H$ is unreasonable in the following circumstance:
a. There is no response to this proposal that is optimal for the insurer, relative to some belief, and which gives type $H$ at least the equilibrium utility.

b. If the insurer’s beliefs after the deviating proposal are that the type is low, and the insurer’s response to this belief is rational, then the low type gets a higher utility.

c. A signalling game is a finite extensive form game in which the privately informed sender chooses a message (that affects payoffs, so this is not cheap talk) after which the uninformed receiver chooses an action. A signalling game has an equilibrium satisfying the intuitive criterion because we can look at trembles for the agent normal form that assign very small probabilities to messages when the sender’s type cannot possibly get the equilibrium expected payoff by playing the message.

3. The only equilibrium of the insurance game satisfying the intuitive criterion is the separating equilibrium in which the low type gets the best (for her) policy among those that would not make the high type better off and which are no worse than actuarially fair for the insurer, relative to $\pi_L$. Denote this policy by $\psi^*_L$.

G. Competition Among Insurers

1. With multiple possible insurers in competition with each other there is an additional equilibrium condition: it is impossible to offer a new contract and make positive profits when those who buy it are those it makes better off.

2. In equilibrium there cannot be any contracts that make positive profits, because otherwise cream skimming would be possible: a new entrant could offer a nearby contract that was more attractive to type
L, but less attractive to type H.

3. For the same reason the equilibrium cannot be pooling.
   a. If there is an equilibrium, H must be purchasing $\psi_H^*$. 

4. In equilibrium type L must be able to buy $\psi_L^*$.
   a. Otherwise a new entrant could attract all type L’s to a policy that was close to $\psi_L^*$ and make slight positive profits.

5. Whether $(\psi_L^*, \psi_H^*)$ is an equilibrium depends on $\alpha$. If $\alpha$ is close to 1 it is not an equilibrium because an entrant could offer a policy that made both types better off and made positive profits.