Answer Key for Problem Set 0  
Econ 4113  
Instructor: Andy McLennan  
Due: The Beginning of Time

**Problem 1:** For the maximization problem

\[
\max U(x_1, x_2) := x_1 x_2 \quad \text{subject to} \quad x_1 + x_2 \leq 2, \ x_1, x_2 \geq 0:
\]

(a) Is it legitimate to assume that \( x_1 + x_2 = 2? \)

(b) Substitute to reduce to a one variable problem.

(c) Solve.

*Answer:*

(a) Yes, since the utility function is monotonic increasing: if \( x'_1 \geq x_1 \) and \( x'_2 \geq x_2 \), then

\[
U(x'_1, x'_2) = (x'_1 - x_1)(x'_2 - x_2) + (x'_1 - x_1)x_2 + x_1(x'_2 - x_2) + U(x_1, x_2) \leq U(x_1, x_2).
\]

(b) Setting \( x_2 = 2 - x_1 \), we obtain the problem

\[
\max x_1(2 - x_1) \quad \text{subject to} \quad 0 \leq x_1 \leq 2.
\]

(c) Setting \( 0 = \frac{d[x_1(2-x_1)]}{dx_1} = 2 - 2x_1 \), we find that the unique critical point of the objective function is \( x_1 = 1 \). The only possibilities for a maximum are \( x_1 = 0, x_1 = 1, \) and \( x_1 = 2 \). Substitution shows that \( x_1 = 1 \) gives the largest value, so the solution to the original problem is

\[
(x_1, x_2) = (1, 1).
\]