Let $C \subset \mathbb{R}^m$ be convex, let $f : C \to \mathbb{R}$ be a concave function, and let $x_0$ be an element of $C$. A subgradient of $f$ at $x_0$ is a linear function $L : \mathbb{R}^m \to \mathbb{R}$ such that

$$f(x) \leq f(x_0) + L(x - x_0)$$

for all $x \in C$.

**Problem 1:** Prove that if $x_0$ is an interior point of $C$, then there is a subgradient of $f$ at $x_0$.

**Problem 2:** Give an example showing that there can be more than one subgradient of $f$ at $x_0$.

**Problem 3:** Give an example showing that there may not be a subgradient of $f$ at $x_0$ if $x_0$ is not an interior point of $C$. 